A Conceptual View of the Doppler Shift

Aki Anandam

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1 Introduction

In this paper, we will review a conceptual view and derivation of the Doppler Shift. It occurs when a source is emitting a wave and translating towards or away from a receiver at a fixed rate. The result is an increase or decrease in the frequency of the wave being emitted.

2 A One Dimensional Case

We will consider the one dimensional case for simplicity, however the ideas presented easily carry over so long as we examine the component of velocity moving towards or away from the receiver. First, we'll imagine the source isn't moving. Then, the waves it emits might look something like [A] to the receiver.

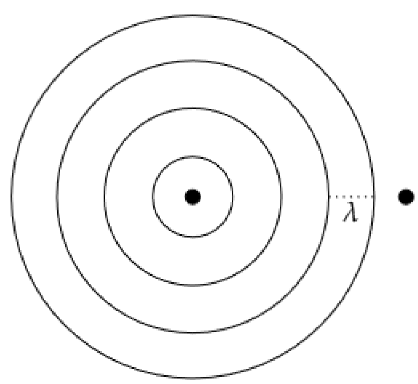

Figure [A]: A stationary source

In this case, the wavelength, or the distance between successive peaks, is λ. λ is constant regardless of where the receiver is placed. Now, we'll examine the case where the source is moving depicted in [B]. The wave emitted will be of the exact same speed and period to the source as the other wave, which should yield the same wavelength. However, this time, the wavelength varies depending on where the receiver is. The source is moving to the right, and we have one receiver to its right and one receiver to its left.

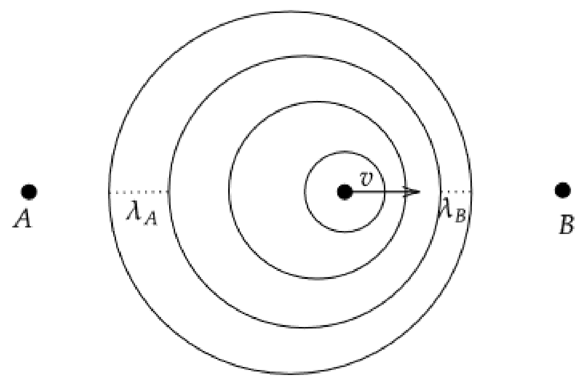

Figure [B]: A moving source

The receiver to the left is A, and its perceived wavelength is λ_A. Likewise, the receiver to the right is B, and its perceived wavelength is λ_B. The source is moving to the right with velocity v. One helpful way to interpret [B] is to think that the source emits a wave this instant, and last instant it had emitted another wave which would have travelled the wavelength. However, last emission, the source was also behind where it is now. So although the wave would have travelled a certain distance, the new wave being emitted will start out ahead of where the last one was. this causes a distortion to the perceived wavelength of the waves.

As a result, λ_B shrinks by vt, where t is the period, because that is the distance the source would have travelled in the time between emissions. So

$$\lambda_B = \lambda - vt. \tag{1}$$

Likewise, λ_A must've increased by vt because the source is moving away from A, so

$$\lambda_A = \lambda + vt. \tag{2}$$

If the speed of the wave is c (it doesn't necessarily have to be a light wave), then

$$t = \frac{\lambda}{c}. \tag{3}$$

We can input (3) into (1) and (2) to say that

$$\lambda_B = \lambda \left(1 - \frac{v}{c}\right) \tag{4}$$

and

$$\lambda_A = \lambda \left(1 + \frac{v}{c}\right). \tag{5}$$

Given that the general frequencies, f, f_A, and f_B are

$$f = \frac{c}{\lambda} \tag{6}$$

$$f_A = \frac{c}{\lambda_A} \tag{7}$$

$$f_B = \frac{c}{\lambda_B}, \tag{8}$$

we can use (4), (5), (6), (7), and (8) to say

$$f_A = \frac{f}{1 + \frac{v}{c}} \tag{9}$$

$$f_B = \frac{f}{1 - \frac{v}{c}}. \tag{10}$$

We can then Taylor expand (9) and (10), only considering velocities where v is small compared to c, to say

$$f_A = f \left(1 - \frac{v}{c}\right) \tag{11}$$

$$f_B = f \left(1 + \frac{v}{c}\right). \tag{12}$$

Like we said, these values aren't very accurate as v approaches c. We have reached the equations for the Doppler Shift, and they are useful in many branches of physics such as astronomy, where they are used to determine, of many things, the speeds and orbits of certain binary stars.

References

[1] Walter Lewin, Doppler Effect, Binary Stars, Neutron Stars and Black Holes, 1999, https://www.youtube.com/watch?v=313C1zo9pyE